

000
001
002
003
004
005
006
007054
055
056
057
058
059
060
061

Polyblur: Removing mild blur by polynomial reblurring Supplementary Material

008
009
010
011
012
013062
063
064
065
066
067

Anonymous CVPR 2021 submission

014
015
016
017068
069
070
071

Paper ID 4417

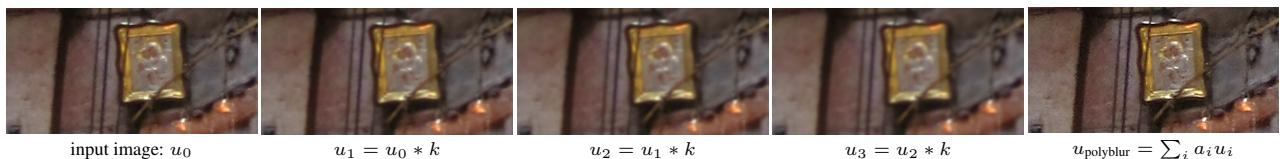
072
073
074

Figure 1: Polyblur removes mild blur by combining multiple applications of the estimated blur.

018
019
020075
076
077

The core idea in our polynomial deblurring algorithm (polyblur) is to combine multiple applications of the estimated blur kernel in a way that allows us to approximate the inverse. An example of this is shown in Figure 1.

078
079
080
081
082
083
084
085021
022
023
024
025

1. Blur model validation and calibration

086
087026
027
028
029
030
031088
089
090
091
092
093
094
095
096
097

The Gaussian blur estimation presented in Section 4 is based on computing gradient features. The gradient feature f_θ is defined as,

$$f_\theta = \min_{\psi \in [0, \pi)} f_\psi = \min_{\psi \in [0, \pi)} \max_{\mathbf{x}} |\nabla_\psi v(\mathbf{x})|, \quad (1)$$

032
033
034
035
036
037098
099
100
101
102
103
104
105
106
107

where $v(\mathbf{x})$ is the input image, and $\nabla_\psi f(\mathbf{x})$ is the directional derivative of v at direction ψ . Then, the strength of the blur is computed by,

$$\sigma_0 = \sqrt{\frac{c^2}{f_\theta^2} - \sigma_b^2}, \quad \sigma_1 = \sqrt{\frac{c^2}{f_{\theta \perp}^2} - \sigma_b^2}. \quad (2)$$

038
039
040
041
042
043
044
045
046
047
048099
100
101
102
103
104
105
106
107

To calibrate c and σ_b , we proceed as follow. Given a set of 50 sharp high quality images, we simulate $K = 1000$ random Gaussian blurry images, by randomly sampling the blur space and the image set. The Gaussian blur kernels are generated by sampling random values for $\sigma_0 \in [0.3, 4]$ and $\rho \in [0.15, 1]$. Additive Gaussian white noise of standard deviation 1% is added to each simulated blurry image.

049
050
051
052
053102
103
104
105
106
107

Examples of simulated Gaussian blur kernels are shown in Figure 2. For each of the blurry images we compute the gradient features according to (1). The parameters c and σ_b are estimated by minimizing the mean absolute error (MAE). The calibrated parameters are $c = 89.8$ and

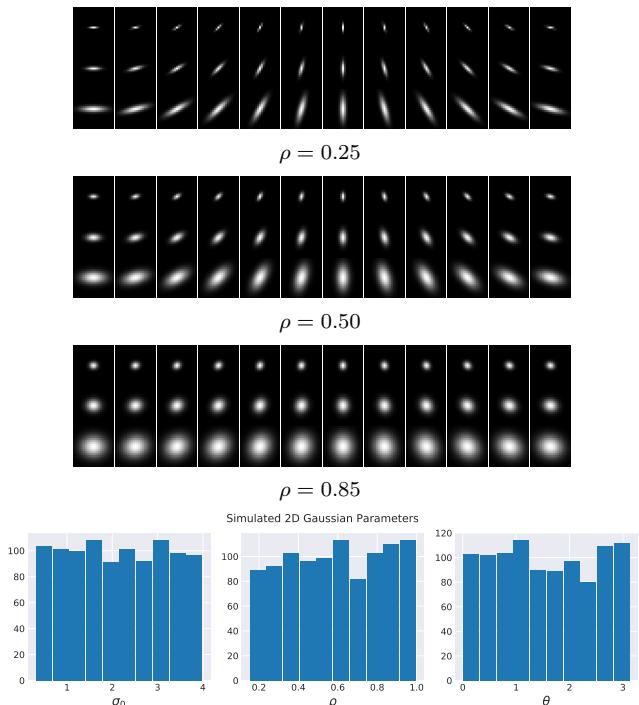


Figure 2: Gaussian Blur kernel examples and distribution of simulated parameters.

$\sigma_b = 0.764$. Note that the values of c and σ_b are implementation dependent (e.g., the finite difference scheme used to compute image gradient).

Figure 3-left shows the relation between the inverse of the estimated gradient feature (i.e., $1/f_\theta$) and the simulated blur kernel σ_0 value. Each of the blur points represents one

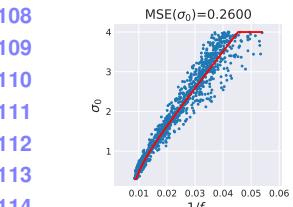


Figure 3: Gaussian blur model calibration. Error in the estimation of σ_0 and σ_1 (and ρ).

simulated image. As we can see, sharp images that have very low blur values ($\sigma_0 \ll 1$) have very similar feature values. The same analysis holds for the gradient feature at the orthogonal direction, and its relation to $\sigma_1 = \rho\sigma_0$ (Figure 3-middle). This validates our *Assumption 1*:

Assumption 1. *In a sharp image, the maximum value of the image gradient in any direction is mostly constant and roughly independent of the image.*

Additionally, Figure 3 shows the almost linear relation (with some spread) between the inverse of the gradient feature $1/f_\theta$ and the blur level σ_0 . This is exactly *Assumption 2*:

Assumption 2. *If a sharp image is affected by Gaussian blur, the blur level in the direction of the principal axis will be linearly related to the inverse of the maximum image gradient in the principal directions.*

Our model contemplates for slight blur that the gradient operator may have introduced when computing the gradient features (σ_b). This is further analyzed at the end of this section for a synthetic image.

In Figure 3-right we show a plot of the real ρ value of the simulated blur kernel and the estimated one. Although there are some outliers, the estimation is in general close to the real value ($MSE(\rho) = 0.121$).

Estimation of blur direction θ . The estimation of the blur direction is done by computing the angle θ with minimum gradient feature value in Eq. (1). Figure 4 shows the error on the estimation of the angle for each simulated blur. As we can see, the error in the angle is quite low for blur kernels highly directional ($\rho < 0.5$). For large values of ρ (e.g., $\rho \in [0.75, 1.0]$), the kernels are almost isotropic, and the estimation is inaccurate. Nevertheless, being almost isotropic, the kernel shape is not affected by the angle value in this case.

Error metrics on estimated kernel values. The ultimate goal in blur estimation is to estimate a blur that is close to the real one. In Figure 5 we present the distribution of two different error metrics that evaluate the distance between the estimated kernel and the simulated one directly on the

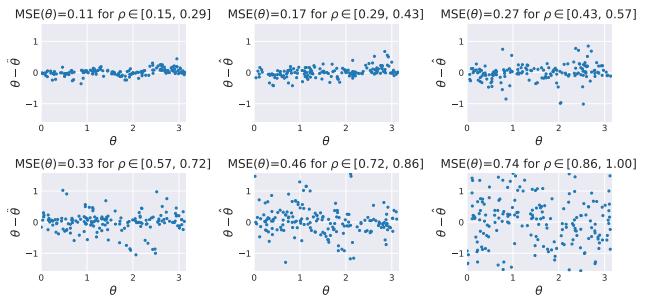


Figure 4: Blur model calibration. Error in estimated parameters. Calibrated parameters are $c = 89.8$, $\sigma_b = 0.764$.

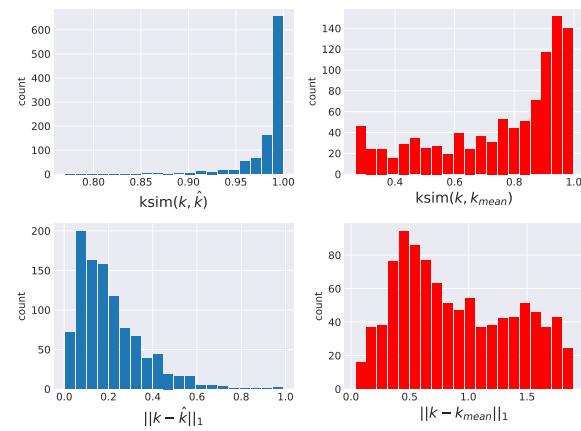


Figure 5: Kernel error metrics on the simulated data. Top row shows the histogram of the kernel similarity between estimated kernels and the respective ground-truth one (left), and between the ground-truth kernels and a fixed isotropic Gaussian kernel (denoted by k_{mean}) having standard deviation the mid-value on the simulated range (right). Bottom row shows the histogram of the ℓ_1 difference between the estimated kernels and the respective ground-truth one (left), and between the ground-truth kernels and k_{mean} .

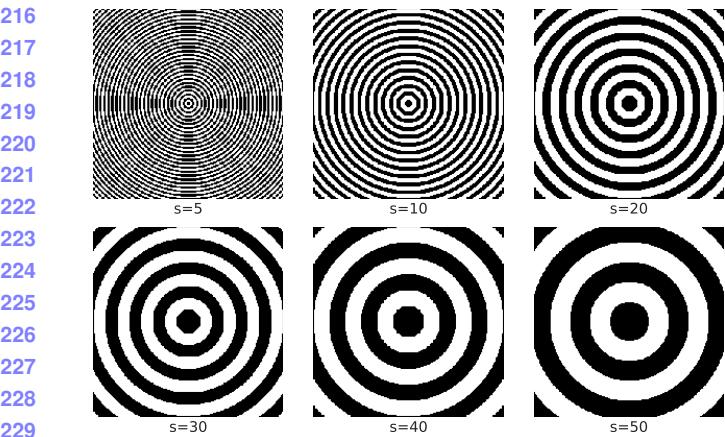
kernel space. The first metric is the kernel similarity, which is the normalized cross correlation:

$$\text{ksim}(k, \hat{k}) = \frac{1}{\|k\| \|\hat{k}\|} \sum_i k_i \hat{k}_i. \quad (3)$$

When both kernels are equal the kernel similarity is 1. The second metric we compute is the ℓ_1 norm between estimated kernel values. To give an idea of the range of both metrics, we also present the error distribution of the real kernel and an average kernel¹. This shows the estimations is accurate.

Concentric circles: A synthetic example. We generated a family of synthetic images with concentric circles at different distances (controlled by a parameter $s \in [5, 50]$), see

¹The average kernel is defined as the isotropic kernel having standard deviation the central value of the simulated range [0.35, 4.0].



(a) Concentric circles pattern.

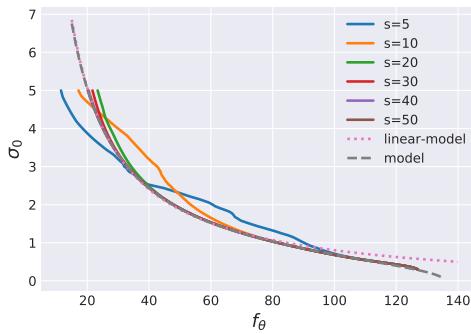
(b) Gradient feature f_θ as a function of simulated blur level σ_0 .

Figure 6: Blur model on a synthetic image.

Figure 6a and blurred them with isotropic Gaussian blur of intensity ($\sigma_0 \in [0.3, 5]$). In Figure 6b we show the computed gradient feature (f_θ) for the the different blurry images (different σ_0) on each simulated pattern ($s = 5$ to $s = 50$). We calibrated the blur model (i.e., estimated c and σ_b values) using the $s = 50$ synthetic pattern image. The model is highly accurate when the circular rings are separated enough so the image can be locally considered a step-edge ($s \geq 20$). However, when the concentric circles are very close (e.g., $s = 5$), the model is not accurate and the blur estimation is biased.

For comparison purposes we also fit a purely linear model that directly maps the inverse of the gradient feature to the sigma value. The linear model (which is the proposed model with $\sigma_b = 0$) is very close to the proposed model except in low sigma values. This is because the proposed model takes into account the (very little) blur σ_b that the gradient operator introduces, leading to a more precise estimation.

2. Blur estimation implementation details

In Algorithm 1 we present the pseudo-code for our estimation of Gaussian blur. We follow Section 4 to estimate

Algorithm 1: Gaussian blur estimation.

input : image u , q , n_{angles} , σ_{\max} , σ_{\min} , ρ_{\max} , ρ_{\min}
output: Blur parameters: σ_0 , ρ , θ .

// 1. Compute gradient features;

```

 $n = \text{Normalize}(u, q);$ 
 $u_x, u_y = \text{ComputeGradient}(n, \sigma_{\max});$ 
for  $i \in [0, n_{\text{angles}}]$  do
     $\Psi_i = i \cdot \pi / n_{\text{angles}};$ 
     $u_{\Psi_i} = u_x \cdot \cos(\Psi_i) - u_y \cdot \sin(\Psi_i);$ 
     $f_{\Psi_i} = \text{Max}(|u_{\Psi_i}|);$ 
end
```

$I_\Psi = \text{Interpolate}(\{f_\Psi\});$

$f_\theta, \theta_0 = \text{Min}(I_\Psi);$

$f_{\theta_\perp}, \theta_{0\perp} = \text{Ortho}(f_\theta, \theta_0);$

// 2. Compute and clamp Gaussian;

$\sigma_0, \sigma_1 = \text{Regression}(f_\theta, f_{\theta_\perp}) // \text{From Eq. (2);}$

$\text{Clamp}(\sigma_0, \sigma_1, \sigma_{\max}, \sigma_{\min});$

$\rho = \sigma_0 / \sigma_1;$

$\text{Clamp}(\rho, \rho_{\min}, \rho_{\max});$

the blur. As the first step the input image is normalized using quantiles ($q = 0.0001$ and $1 - q$) to be robust to outliers. From the image gradient (u_x, u_y) we compute the directional derivative at n_{angles} (usually $n_{\text{angles}} = 6$) uniformly covering $[0, \pi)$. The maximum direction of the magnitude for each sample angle is found. Among the n_{angles} maximum values, we find the minimum value and angle (f_0, θ_0) through bicubic interpolation. Using Eq. 2 we compute σ_0 and σ_1 (and compute $\rho = \sigma_1 / \sigma_0$).

The gradient features can be efficiently computed in parallel not just between different angles but also between each different pixel. Computing the maximum can be represented as a gather operation that can be optimized using shared memory and tiling. Within our pipeline this represents around 40 – 53% of the computation, this comes from the fact that we compute a maximum magnitude n_{angles} times.

3. Comparison on DIV2K dataset

In figures 7–11 we present additional results to the one presented on Section 6. We generated a mild-blur dataset by artificially blurring sharp images from the DIV2K dataset [1]. Each of the 100 images in the validations dataset were blurred by a random Gaussian blur kernel of different sizes, shapes, and orientations (i.e., $\sigma_0 \sim \mathcal{U}[0.3, 4]$, $\rho \sim \mathcal{U}[0.15, 1.0]$, $\theta \sim \mathcal{U}[0, \pi]$). Additive white Gaussian noise of standard deviation 1% was added on top. We compared Polyblur (one, two, and three iterations) to the following adaptive sharpening or deblurring methods: SRN-Deblur [9], Sparse Deblurring [10], DeblurGANv2

324 (inception and mobilenet architectures) [6], Spectral Irreg-
325 ularities [3], L0-Deblur [8], GLAS [11], Guided Filter [4],
326 Convolutional Deblurring [5].
327

328 4. Deblurring before super-resolution

329 Figures 12 and 13 present additional results to Section 6
330 on deblurring before super-resolution. We apply Polyblur
331 as a pre-step before using an off-the-shelf deep network
332 for doing $4\times$ image upscaling. We trained from scratch
333 an EDSR [7] network with 32 layers and 64 filters using
334 DIV2K training dataset. Polyblur produces the best quanti-
335 tative and qualitative results. The evaluation is done on the
336 DIV2KRK dataset introduced in [2].
337

338 5. Comparison on images in the wild.

339 In figures 14 to 18 we present additional results of Poly-
340 blur applied to some images in the wild (Section 6). We
341 compare Polyblur (one, two and three iterations) to the fol-
342 lowing adaptive sharpening or deblurring methods: SRN-
343 Deblur [9], Sparse Deblurring [10], DeblurGANv2 (incep-
344 tion and mobilenet architectures) [6], Spectral Irregulari-
345 ties [3], L0-Deblur [8], GLAS [11], Guided Filter [4], Con-
346 volutional Deblurring [5]. Polyblur manages to remove
347 mild blur, as the one present in most images, without in-
348 troducing any new artifacts.
349

350 351 References

- 352 [1] Eirikur Agustsson and Radu Timofte. Ntire 2017 challenge
353 on single image super-resolution: Dataset and study. In *The*
354 *IEEE Conference on Computer Vision and Pattern Recog-*
355 *nition (CVPR) Workshops*, July 2017. 3
- 356 [2] Sefi Bell-Kligler, Assaf Shocher, and Michal Irani. Blind
357 super-resolution kernel estimation using an internal-gan. In *Advances in Neural Information Processing Systems*, pages
358 284–293, 2019. 4, 10, 11
- 359 [3] Amit Goldstein and Raanan Fattal. Blur-kernel estimation
360 from spectral irregularities. In *European Conference on*
361 *Computer Vision*, pages 622–635. Springer, 2012. 4
- 362 [4] Kaiming He, Jian Sun, and Xiaou Tang. Guided image fil-
363 tering. *IEEE transactions on pattern analysis and machine*
364 *intelligence*, 35(6):1397–1409, 2012. 4
- 365 [5] Mahdi S Hosseini and Konstantinos N Plataniotis. Convolu-
366 tional deblurring for natural imaging. *IEEE Transactions on*
367 *Image Processing*, 29:250–264, 2019. 4
- 368 [6] Orest Kupyn, Tetiana Martyniuk, Junru Wu, and Zhangyang
369 Wang. Deblurgan-v2: Deblurring (orders-of-magnitude)
370 faster and better. In *Proceedings of the IEEE International*
371 *Conference on Computer Vision*, pages 8878–8887, 2019. 4
- 372 [7] Bee Lim, Sanghyun Son, Heewon Kim, Seungjun Nah, and
373 Kyoung Mu Lee. Enhanced deep residual networks for single
374 image super-resolution. In *Proceedings of the IEEE confer-*
375 *ence on computer vision and pattern recognition workshops*,
376 pages 136–144, 2017. 4

- 377 [8] Jinshan Pan, Zhe Hu, Zhixun Su, and Ming-Hsuan Yang. l_0 -
378 regularized intensity and gradient prior for deblurring text
379 images and beyond. *IEEE transactions on pattern analysis*
380 *and machine intelligence*, 39(2):342–355, 2016. 4
- 381 [9] Xin Tao, Hongyun Gao, Xiaoyong Shen, Jue Wang, and Ji-
382 aya Jia. Scale-recurrent network for deep image deblurring.
383 In *Proceedings of the IEEE Conference on Computer Vision*
384 *and Pattern Recognition*, pages 8174–8182, 2018. 3, 4
- 385 [10] Haichao Zhang, David Wipf, and Yanning Zhang. Multi-
386 image blind deblurring using a coupled adaptive sparse prior.
387 In *Proceedings of the IEEE Conference on Computer Vision*
388 *and Pattern Recognition*, pages 1051–1058, 2013. 3, 4
- 389 [11] Xiang Zhu and Peyman Milanfar. Restoration for weakly
390 blurred and strongly noisy images. In *2011 IEEE Workshop*
391 *on Applications of Computer Vision (WACV)*, pages 103–
392 109. IEEE, 2011. 4

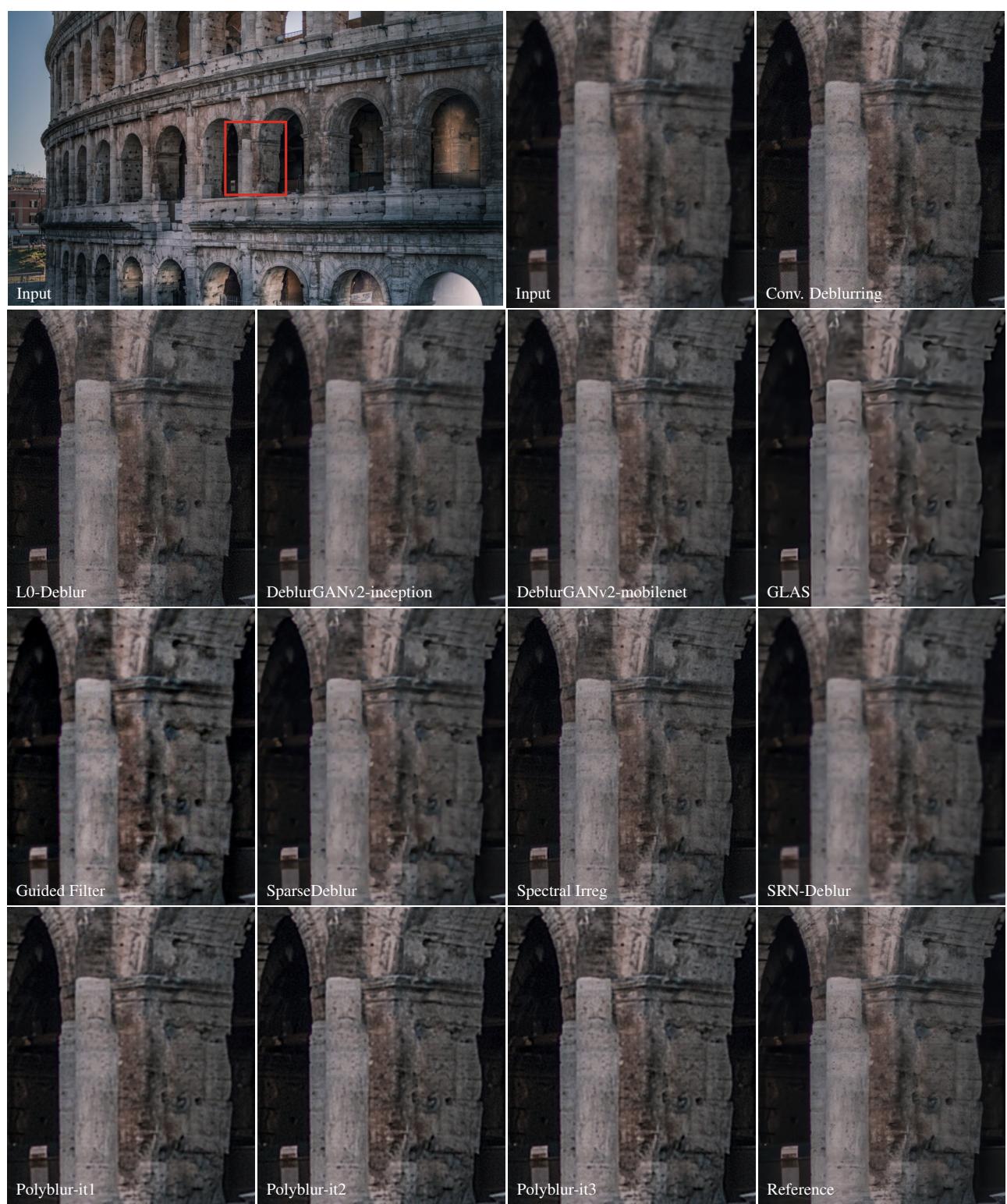


Figure 7: Example of comparison on one image from the DIV2K validation dataset with synthetic blur.

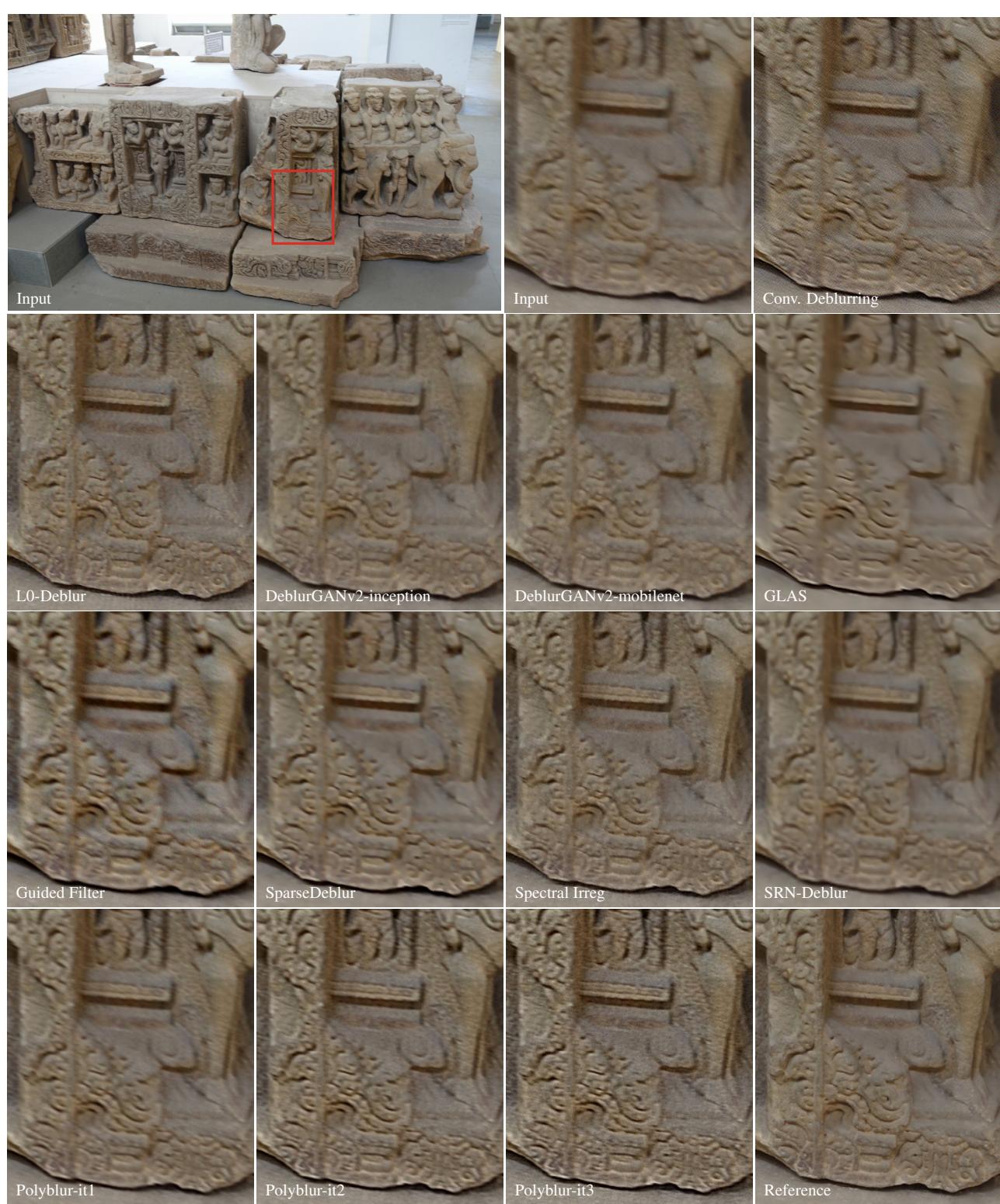


Figure 8: Example of comparison on one image from the DIV2K validation dataset with synthetic blur.

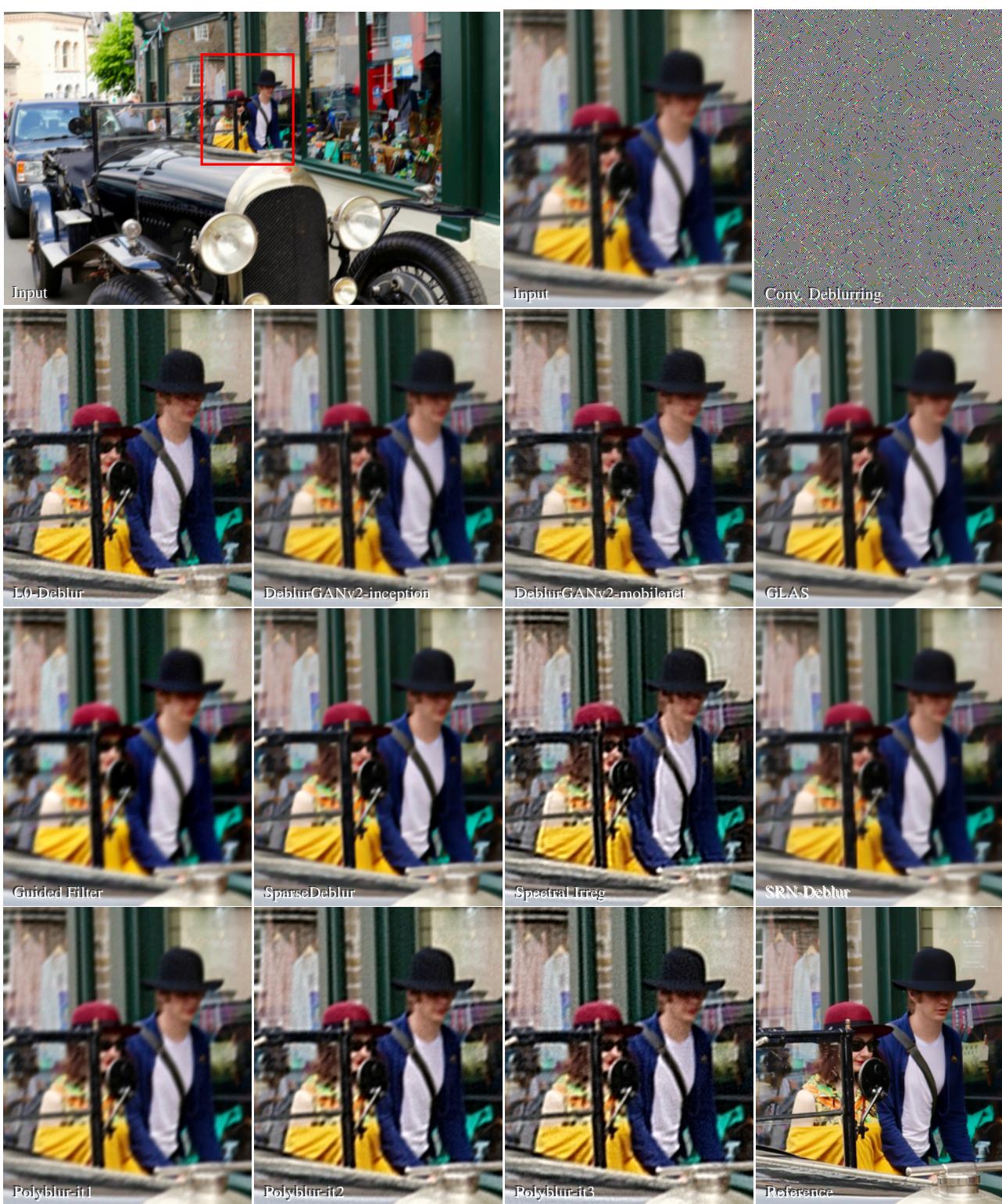


Figure 9: Example of comparison on one image from the DIV2K validation dataset with synthetic blur.



Figure 10: Example of comparison on one image from the DIV2K validation dataset with synthetic blur.

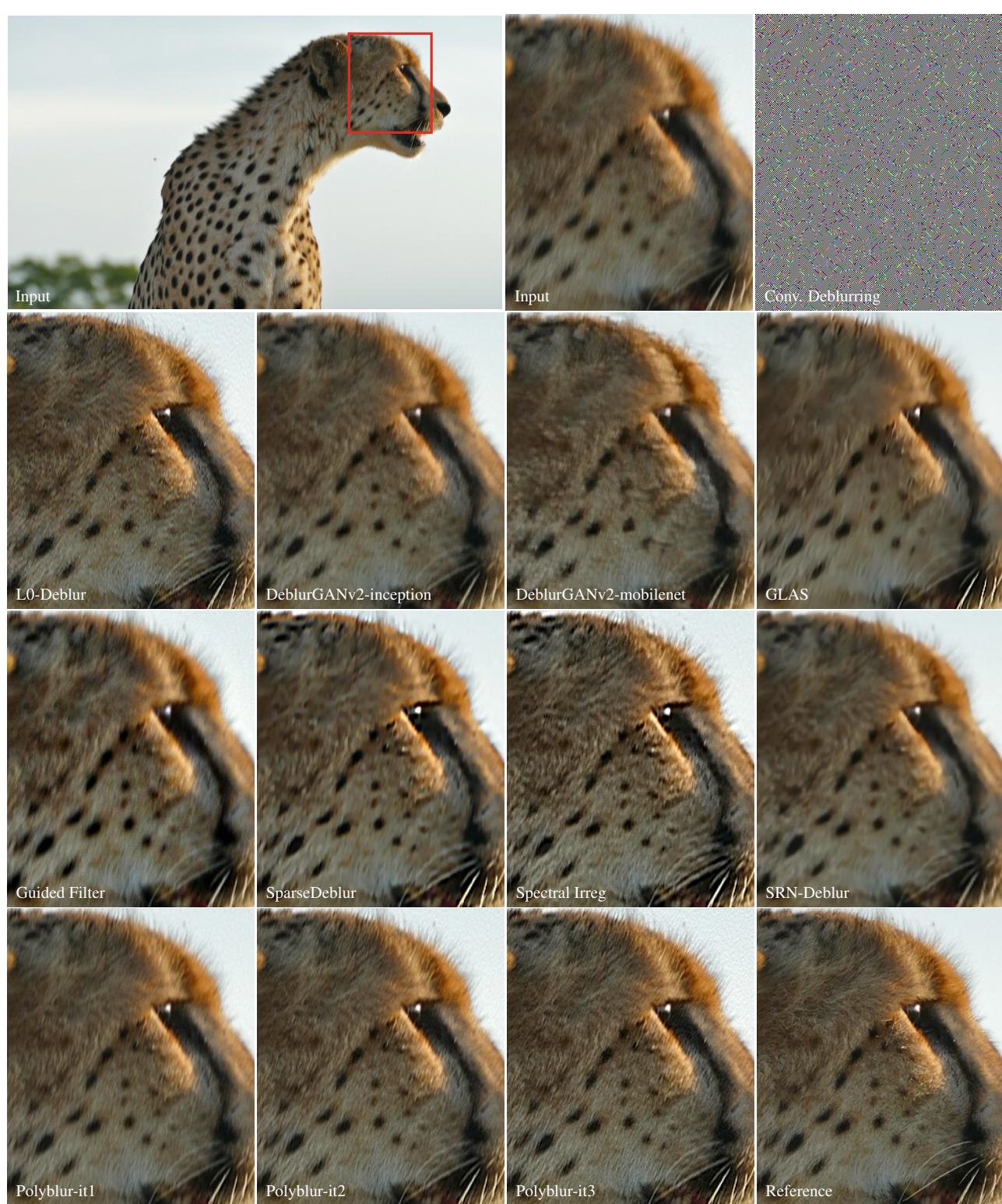
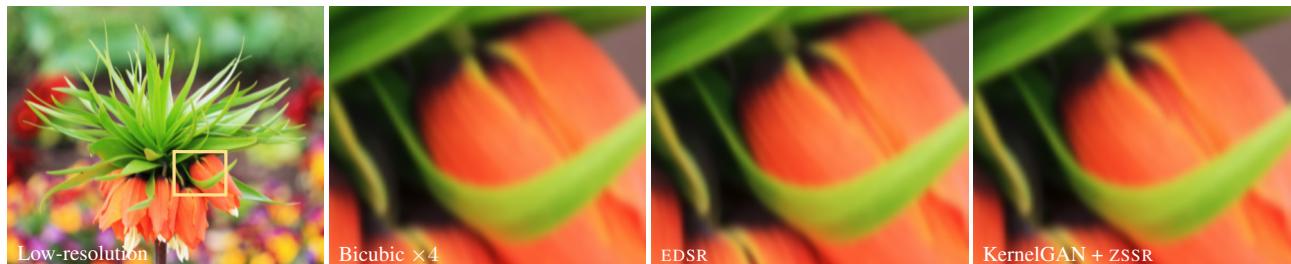
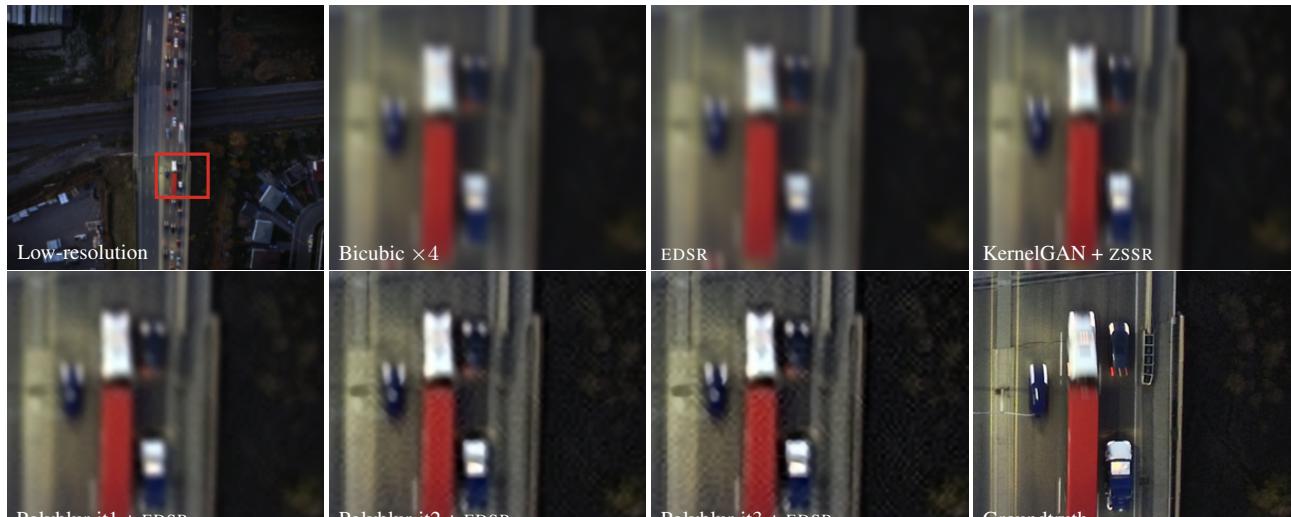
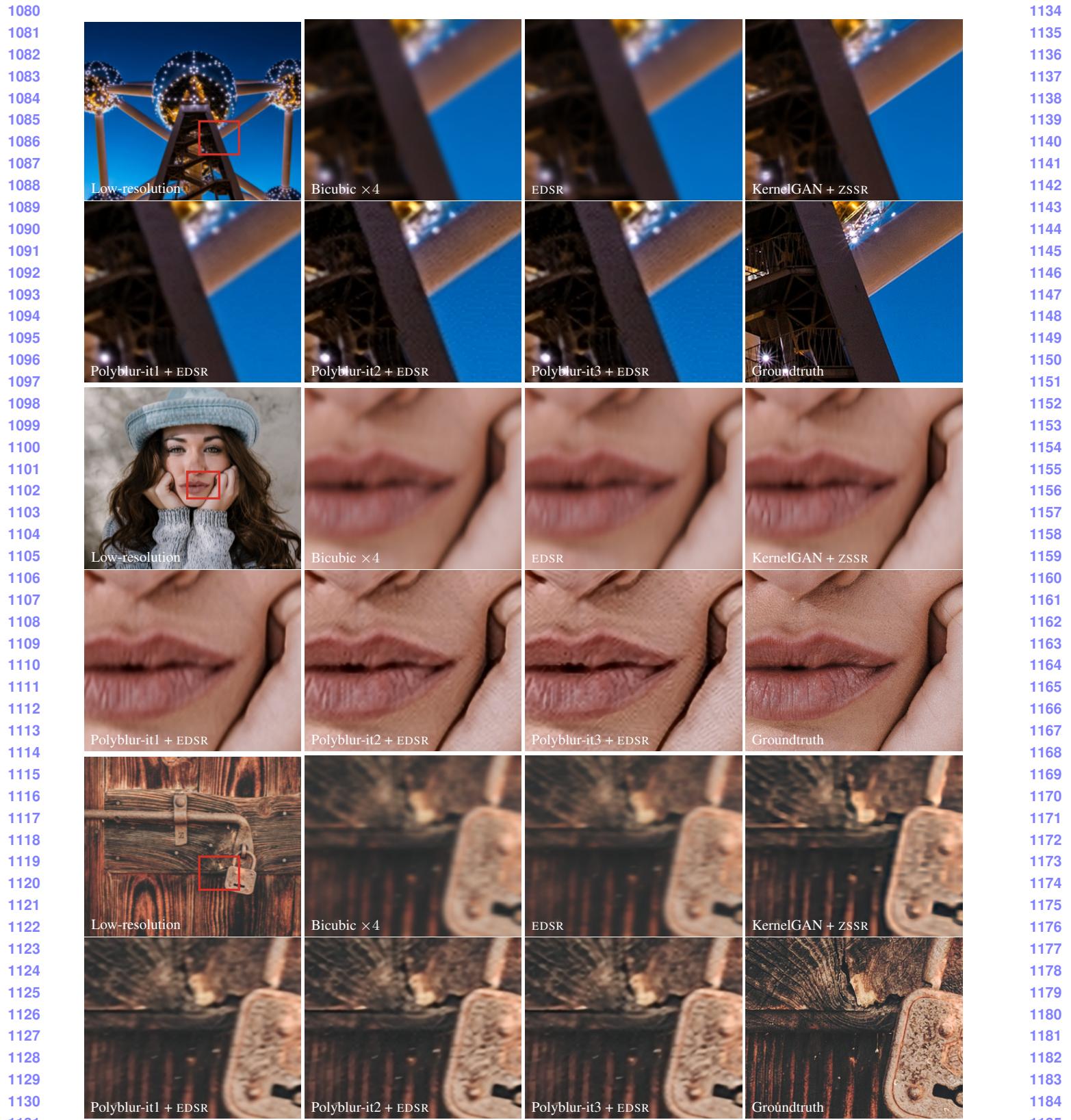
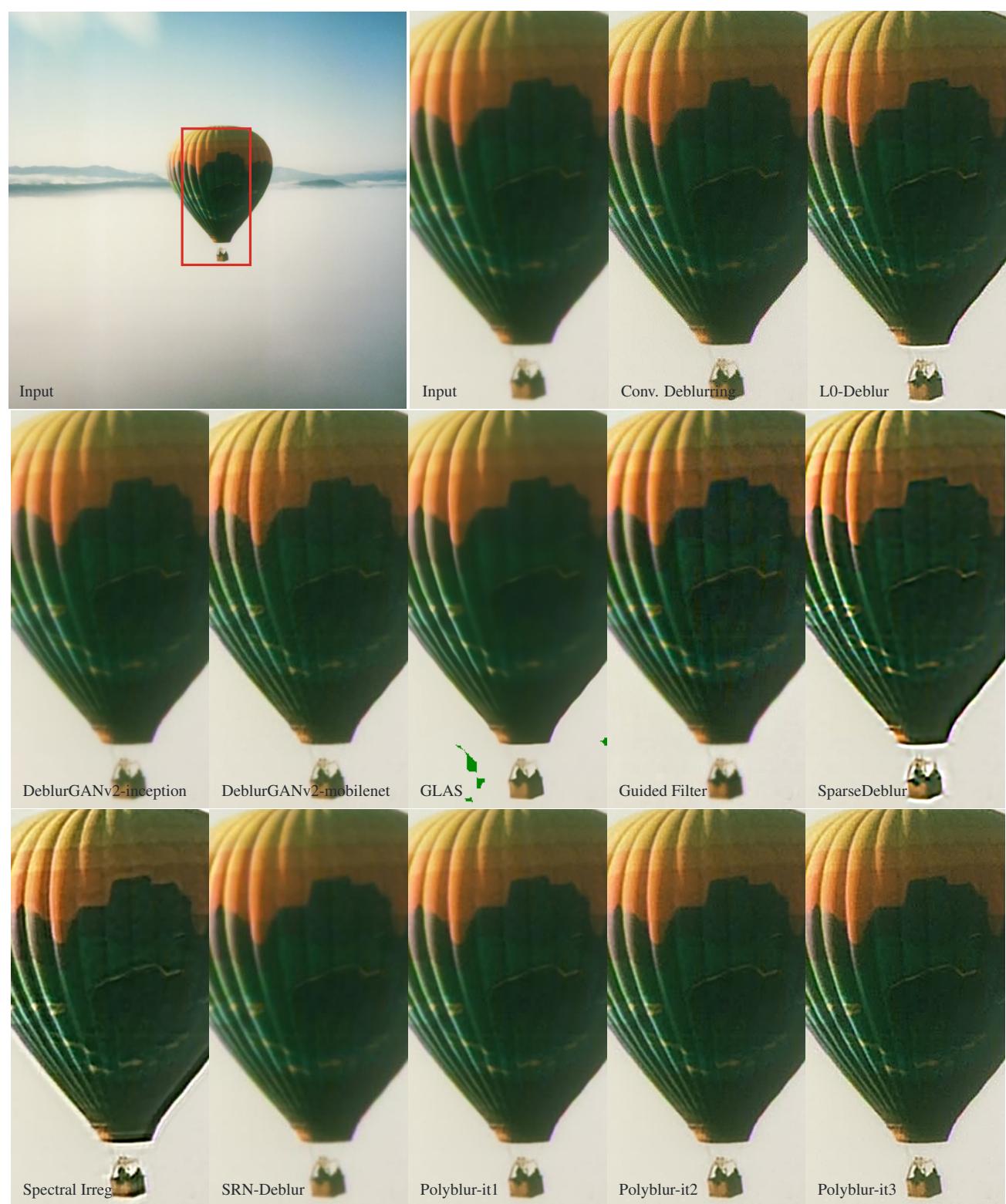


Figure 11: Example of comparison on one image from the DIV2K validation dataset with synthetic blur.

972
973
974
975
976
977
978
979
980
981
982
983
984
985
986
987
988
9891026
1027
1028
1029
1030
1031
1032
1033
1034
1035
1036
1037
1038
1039
1040
1041
1042
1043990
991
992
993
994
995
996
997
998
999
1000
1001
1002
1003
1004
1005
10061044
1045
1046
1047
1048
1049
1050
1051
1052
1053
1054
1055
1056
1057
1058
1059
10601061
1062
1063
1064
1065
1066
1067
1068
1069
1070
1071
1072
1073
1074
1075
1076
1077
1078
1079Figure 12: DIV2KRK [2] 4× upscaling with *unknown* kernel and out-of-the-shelf super-resolution model.

Figure 13: DIV2KRK [2] 4× upscaling with *unknown* kernel and out-of-the-shelf super-resolution model.

Figure 14: Example of removing mild blur *in the wild*.

1296
1297
1298
1299
1300
1301
1302
1303
1304
1305
1306
1307
1308
1309
1310
1311
1312
1313
1314
1315
1316
1317
1318
1319
1320
1321
1322
1323
1324
1325
1326
1327
1328
1329
1330
1331
1332
1333
1334
1335
1336
1337
1338
1339
1340
1341
1342
1343
1344
1345
1346
1347
1348
1349

1350
1351
1352
1353
1354
1355
1356
1357
1358
1359
1360
1361
1362
1363
1364
1365
1366
1367
1368
1369
1370
1371
1372
1373
1374
1375
1376
1377
1378
1379
1380
1381
1382
1383
1384
1385
1386
1387
1388
1389
1390
1391
1392
1393
1394
1395
1396
1397
1398
1399
1400
1401
1402
1403

Figure 15: Example of removing mild blur *in the wild*.

Figure 16: Example of removing mild blur *in the wild*.

1512
1513
1514
1515
1516
1517
1518
1519
1520
1521
1522
1523
1524
1525
1526
1527
1528
1529
1530
1531
1532
1533
1534
1535
1536
1537
1538
1539
1540
1541
1542
1543
1544
1545
1546
1547
1548
1549
1550
1551
1552
1553
1554
1555
1556
1557
1558
1559
1560
1561
1562
1563
1564
1565

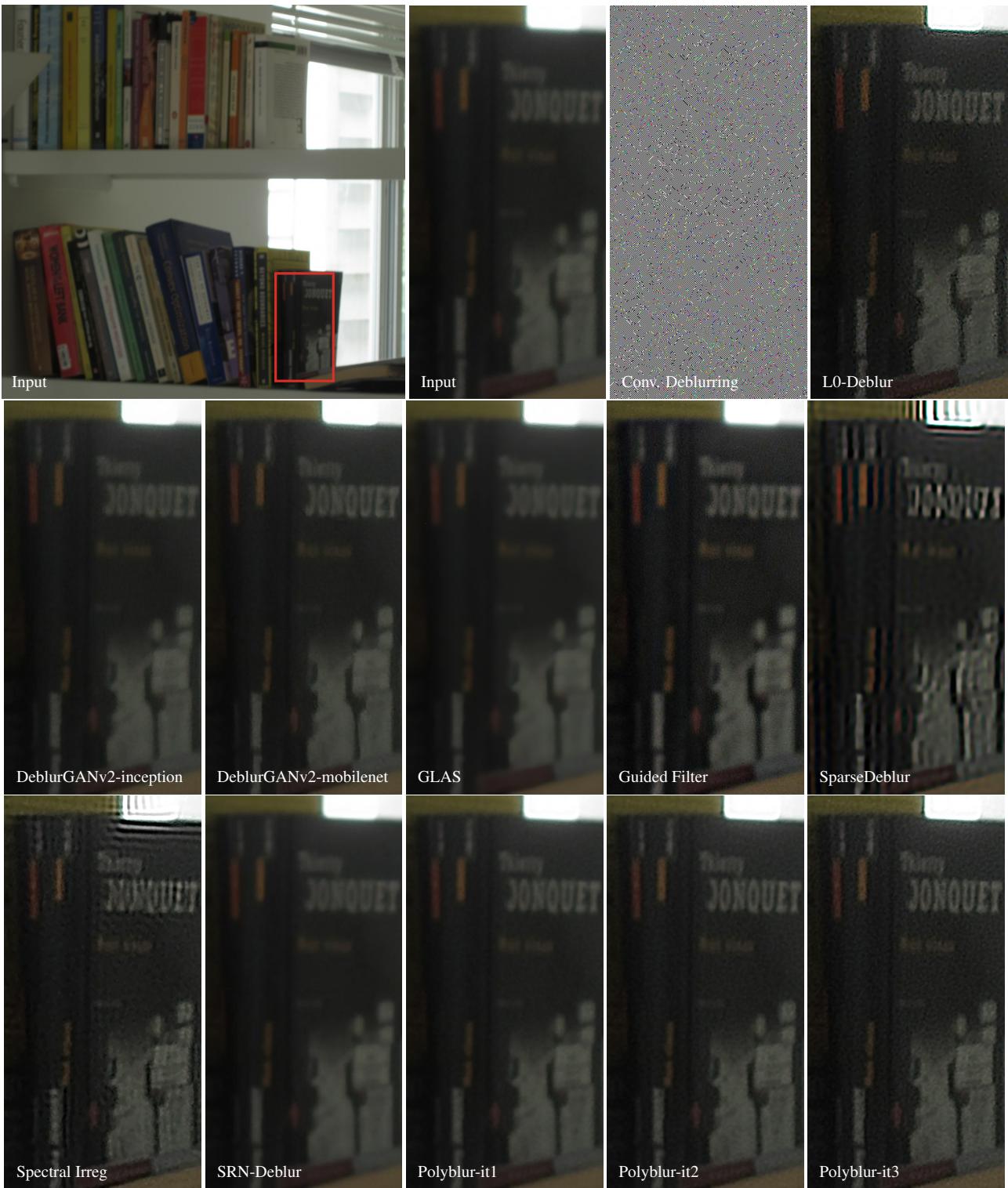
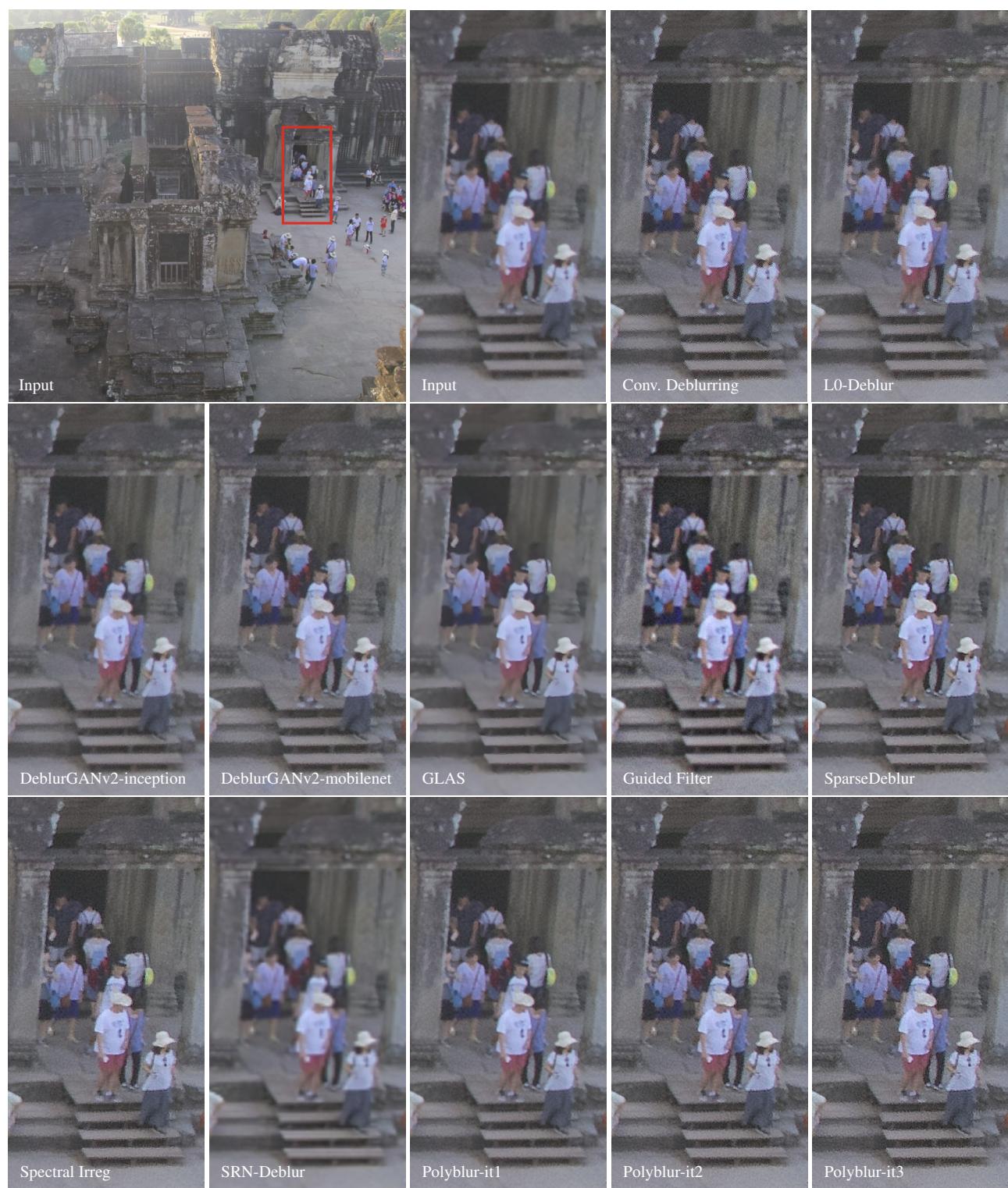


Figure 17: Example of removing mild blur *in the wild*.

CVPR 2021 Submission #4417. CONFIDENTIAL REVIEW COPY. DO NOT DISTRIBUTE.

Figure 18: Example of removing mild blur *in the wild*.