Supplemental Material

A. Full Formulation and Discretization

As an example, we show the full differential formulation of the u wind variable:

Formula: $\frac{Du}{Dt} = -c_{pd}\theta_v \frac{\partial \Pi'}{\partial x} + \kappa \nabla^2 u_{n-1}$ **Material Derivative:** $\frac{Du}{Dt} \equiv \frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{1}{\rho}\frac{\partial \rho uw}{\partial z}$

Expansion: $\frac{\partial u}{\partial t} = -\frac{\partial u^2}{\partial x} - \frac{\partial uv}{\partial y} - \frac{1}{\rho}\frac{\partial \rho uw}{\partial z} - c_{pd}\theta_v \frac{\partial \Pi'}{\partial x} + \kappa_x \frac{\partial^2 u_{n-1}}{\partial x^2} + \kappa_y \frac{\partial^2 u_{n-1}}{\partial y^2} + \kappa_z \frac{\partial^2 u_{n-1}}{\partial z^2}$

Note that ρ is added to the vertical wind component so the discretization uses the correct density. Next, we show its discretization:

$$\begin{aligned}
\frac{\partial \boldsymbol{u}_{(i,j,k)}}{\partial \boldsymbol{t}} = &-\frac{1.0}{\Delta x}\left(\left(\frac{u_{(i+1,j,k)} + u_{(i,j,k)}}{2.0}\right)^2 - \left(\frac{u_{(i,j,k)} + u_{(i-1,j,k)}}{2.0}\right)^2\right) - \frac{1.0}{\Delta y}\left(\left(\frac{u_{(i,j+1,k)} + u_{(i,j,k)}}{2.0}\right)\left(\frac{v_{(i,j+1,k)} + v_{(i-1,j+1,k)}}{2.0}\right) - \left(\frac{u_{(i,j,k)} + u_{(i,j-1,k)}}{2.0}\right)\left(\frac{v_{(i,j,k)} + v_{(i-1,j,k)}}{2.0}\right)\right) \\
&- \frac{1.0}{\rho_{(i,j,k)}\,\Delta z_{(k)}}\left(\left(\frac{\rho_{(i,j,k+1)} + \rho_{(i,j,k)}}{2.0}\right)\left(\frac{u_{(i,j,k+1)} + u_{(i,j,k)}}{2.0}\right)\left(\frac{w_{(i,j,k+1)} + w_{(i-1,j,k+1)}}{2.0}\right)\right. \\
&\left. - \left(\frac{\rho_{(i,j,k)} + \rho_{(i,j,k-1)}}{2.0}\right)\left(\frac{u_{(i,j,k)} + u_{(i,j,k-1)}}{2.0}\right)\left(\frac{w_{(i,j,k)} + w_{(i-1,j,k)}}{2.0}\right)\right) - \frac{1.0}{\Delta x} c_{pd}\theta_{t=0_{(i,j,k)}}\left(1.0 + 0.61\, q_{v_{t=0_{(i,j,k)}}}\right)\left(\Pi'_{(i,j,k)} - \Pi'_{(i-1,j,k)}\right) \\
&+ \kappa_x\left(\frac{u_{n-1_{(i+1,j,k)}} - 2.0\, u_{n-1_{(i,j,k)}} + u_{n-1_{(i-1,j,k)}}}{\Delta x^2}\right) + \kappa_y\left(\frac{u_{n-1_{(i,j+1,k)}} - 2.0\, u_{n-1_{(i,j,k)}} + u_{n-1_{(i,j-1,k)}}}{\Delta y^2}\right) \\
&+ \kappa_z\left(\frac{u_{n-1_{(i,j,k+1)}} - 2.0\, u_{n-1_{(i,j,k)}} + u_{n-1_{(i,j,k-1)}}}{\Delta z_{(k)}{}^2}\right)
\end{aligned}$$

B. Land Use Constants

We show a table of the main land-use constants [Stull 1988; Noilhan and Planton 1989; Oke 2002]:

No	Name	a	C_{GA}	d_s	B_o
1	bare soil	0.25	60.5	9	1.5
2	grass	0.21	114.27	5.9	0.5
3	forest	0.13	101.73	5.9	0.35
4	snow	0.6	10	0.2	0.5
5	crops	0.19	120.54	5.9	0.6
6	urban	0.15	154.11	8.3	3
7	water	0.08	106.66	1	0.1
8	low resid.	0.18	124.11	1	1
9	desert	0.4	41.42	0.01	10
10	low ind.	0.1	164.10	1	3
11	high ind.	0.1	184.10	1	3.5

C. Atmospheric Soundings

Sounding example from the University of Wyoming database for Norman, Oklahoma at 00Z on August 2, 2014.

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ATMOSP                  DEW   REL    MIX   WIND  WIND
 PRES   HGHT   TEMP   POINT  HUMD  RATIO   DIR  SPEED    THETA
  hPa     m      C      C     %     g/kg   deg   knot      K
-----------------------------------------------------------------------------
1000.0   136
 977.0   345   27.6   19.6   62  14.92    75      4   302.8
 965.0   453   25.6   17.6   61  13.30    87      6   301.8
 947.8   610   24.1   16.5   63  12.61   105      9   301.8
 925.0   821   22.0   15.0   64  11.72   105      4   301.8
 915.1   914   21.1   14.7   67  11.60    85      2   301.8
 889.0  1164   18.8   13.8   73  11.28   142      2   301.9
 883.4  1219   19.0   11.2   61   9.53   155      2   302.7
 883.0  1223   19.0   11.0   60   9.42   154      2   302.7
 867.0  1380   17.8    8.8   56   8.26   104      6   303.1
 856.0  1489   18.2    1.2   32   4.90    69      9   304.6
 850.0  1549   17.8   -1.2   27   4.14    50     10   304.8
 837.0  1681   18.0  -11.0   13   1.98    38     13   306.3
 831.0  1742   17.4   -4.6   22   3.28    33     14   306.3
 824.0  1814   16.6    4.6   45   6.49    26     16   306.2
 822.5  1829   16.5    4.8   46   6.61    25     16   306.3
 815.0  1908   16.0    6.0   51   7.24    24     16   306.6
 793.5  2134   14.5    4.5   51   6.71    20     17   307.3
 780.0  2279   13.6    3.6   51   6.39    18     17   307.9
 765.4  2438   12.3    3.4   55   6.41    15     17   308.1
 754.0  2564   11.2    3.2   58   6.43    15     19   308.2
 747.0  2642   11.0    1.0   50   5.54    15     20   308.9
 741.0  2709   10.4    2.4   58   6.18    15     21   308.9
 738.0  2743   10.1    2.2   58   6.13    15     21   308.9
 718.0  2971    8.2    1.2   61   5.85    11     23   309.3
 711.3  3048    7.9   -3.5   44   4.16    10     24   309.8
 700.0  3180    7.4  -11.6   25   2.26    10     18   310.6
 681.0  3405    5.8  -14.2   22   1.88     5     19   311.3
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D. Variables

Variable	Description	Units
U	Wind velocity	Vector (m s^{-1})
θ	Potential temperature	K
Π	Exener function	Non-dimensional
q_v q_c q_r	Mixing rations: vapor, condensation, rain	kg kg^{-1}
$U \cdot \nabla$	Advection	s^{-1} (m/s · 1/m)
κ	Diffusion coefficient	s^{-1} (m/s · 1/m)
D/Dt	Material derivative	s^{-1} (m/s · 1/m)
R_d	Specific heat of dry air at constant pressure	J kg^{-1} K^{-1}
c_{pd}	Heat capacity of	J kg^{-1} K^{-1}
θ	Potential temp.	K
g	Gravity	m s^{-2}
$\theta_{t=0}$	Initial potential temp.	K
ρ	Density	kg m^{-3}
$C_{v\to c}$	Condensation	kg kg^{-1}
$E_{c,r\to v}$	Evaporation	kg kg^{-1}
V_c	Ventilation coefficient	kg kg^{-1}
$A_{c\to r}$	Autoconversion	kg kg^{-1}
$B_{c\to r}$	Accretion	kg kg^{-1}
V_t	Rain terminal velocity	m s^{-1}
$\rho_{z=0}$	Density at z=0	kg m^{-3}
T_z	Temp. at lowest grid cell	K
$p_{z=0}$	Pressure at z=0	kPa
Q^*	Radiation flux	W m^{-2}
$K\uparrow, K\downarrow$	Reflected/Transmitted shortwave solar radiation	W m^{-2}
$I\uparrow, I\downarrow$	Emitted/Diffusive Longwave radiation	W m^{-2}
S_c	Solar radiation	W m^{-2}
T_K	Sky transmissivity	%
Ψ	Solar elevation	Radian
σ_{C_L} σ_M σ_H	Fraction of cloud coverage	%
g_{lat}, g_{long}	Geographic latitude and longitude	Radian
t_{UTC}	Time in UTC	h
δ_s	Solar decline angle	Radian
a	Albedo	%
Q_H, Q_L, Q_G	Sensible/Latent/Ground heat flux	W m^{-2}
C_{GA}	Soil heat capacity per area	J m^{-w} K^{-1}
T_m	Seasonally varying mean temperature	K
T_g	Temp. ground (first d_s cm)	K
a_{FR}	Conductivity air-ground	W m^{-1} K^{-1}
B_o	Bowens ratio	-
T_a	Temp. air above z=0	K
ϕ, ϕ_s	Deterministic, Stochastic ϕ	-
$\Omega = \{w_l, w_p, w_p\}$	Input parameters: land use, procedural, weather	-
$E(*)$	Error function optimization	-
$C(*)$	Cost minimization	-
a_{min}	Acceptance ratio	%
β	Energy level	-
η	Objective value	-
λ	Robert-Asselin filter parameter	-